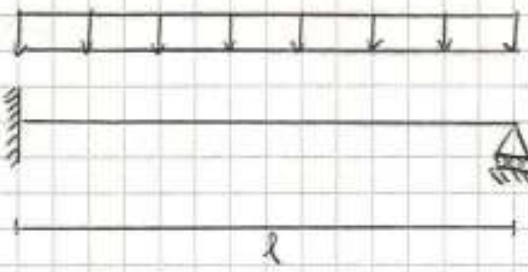
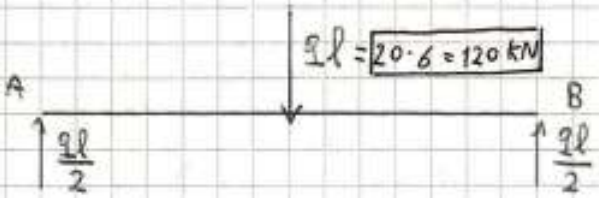
• CONDIZIONI AL CONTORNO



E' IPERSTATICA UNA VOLTA

GDL = 3

GDV = 4



(A) $r(0) = 0$ $\quad \varphi(0) = 0$

(B) $r(l) = 0$ $\quad \varphi(l) \neq 0$ quindi $\Pi(l) = 0 \Rightarrow \frac{d^2 r}{ds^2} = 0 \Rightarrow r'' = 0$

CONDIZIONI IN A

$$r(0) = \frac{q_2}{EI}\frac{s^4}{24} + C_1\frac{s^3}{6} + C_2\frac{s^2}{2} + C_3 s + C_4 = 0$$

$$r(0) = \frac{q_2}{EI}\frac{(0)^4}{24} + C_1\frac{(0)^3}{6} + \frac{C_2}{2}(0)^2 + C_3(0) + C_4 = 0 \Rightarrow \boxed{C_4 = 0}$$

$$\varphi(0) = \frac{dr}{ds} = \frac{q_2}{EI} + \frac{s^3}{6} + C_1\frac{s^2}{2} + C_2 s + C_3 = 0$$

$$\varphi(0) = \frac{q_2}{EI} + \frac{(0)^3}{6} + C_1\frac{(0)^2}{2} + C_2(0) + C_3 = 0 \Rightarrow \boxed{C_3 = 0}$$

CONDIZIONI IN B

$$r(l) = \boxed{\frac{q_2}{EI}\frac{(l)^4}{24} + C_1\frac{(l)^3}{6} + \frac{C_2}{2}(l)^2 + C_3(l) + C_4 = 0}$$

$$M = \frac{d^2 r}{ds^2} = \frac{q_2}{EI}\frac{s^2}{2} + C_1 s + C_2 = 0 \Rightarrow \Pi(l) = \boxed{\frac{q_2}{EI}\frac{(l)^2}{2} + C_1 l + C_2 = 0}$$

Mettiamo a sistema per trovare le incognite C_1, C_2, C_3 e C_4

$$\begin{cases} C_4 = 0 \\ C_3 = 0 \\ -\frac{q_2}{EI}\frac{l^4}{24} + C_1\frac{l^3}{6} + \frac{C_2}{2}l^2 + C_3 l + C_4 = 0 \\ -\frac{q_2}{EI}\frac{l^2}{2} + C_1 l + C_2 = 0 \end{cases} \Rightarrow \begin{cases} C_4 = 0 \\ C_3 = 0 \\ -\frac{q_2}{EI}\frac{l^4}{24} + C_1\frac{l^3}{6} + \frac{C_2}{2}l^2 + (0)l + 0 = 0 \\ -\frac{q_2}{EI}\frac{l^2}{2} + C_1 l + C_2 = 0 \end{cases}$$

$$\begin{cases} C_4 = 0 \\ C_3 = 0 \\ -\frac{q_2}{EI}\frac{l^4}{24} + C_1\frac{l^3}{6} + \frac{C_2}{2}l^2 = 0 \\ -\frac{q_2}{EI}\frac{l^2}{2} + C_1 l + C_2 = 0 \end{cases} \Rightarrow \begin{cases} C_4 = 0 \\ C_3 = 0 \\ -\frac{q_2}{EI}\frac{l^4}{24} + \left(\frac{q_2}{EI}\frac{l}{2} - \frac{C_2}{l}\right)\frac{l^3}{6} + \frac{C_2}{2}l^2 = 0 \\ C_1 = \frac{q_2}{EI}\frac{l^2}{2\cdot 1} - \frac{C_2}{l} \Rightarrow C_1 = \frac{q_2}{EI}\frac{l}{2} - \frac{C_2}{l} \end{cases} \bullet$$

$$\begin{cases} C_4 = 0 \\ C_3 = 0 \\ -\frac{q_2}{EI}\frac{l^4}{24} + \frac{q_2}{EI}\cdot\frac{l^4}{12} - C_2\frac{l^2}{6} + \frac{C_2}{2}l^2 = 0 \ \ast \\ C_1 = \frac{q_2}{EI}\cdot\frac{l}{2} - \frac{C_2}{l} \end{cases}$$

$$\ast \quad -\frac{C_2 l^2 + 3C_2 l^2}{6} - \frac{q_2 l^4 + 2q_2 l^4}{24EI} = 0$$

$$\frac{{}^{1}\!\not{2}\, C_2 l^2}{\not{6}_3} + \frac{q_2 l^4}{24EI} = 0 \Rightarrow C_2\frac{l^2}{3} + \frac{q_2}{EI}\frac{l^4}{24} = 0$$

$$C_2 = -\frac{q_2}{EI}\frac{l^{\not{4}^{\,2}}}{\underset{8}{24}}\cdot\frac{\not{2}}{\not{l}^2} \Rightarrow \boxed{C_2 = -\frac{q_2}{EI}\frac{l^2}{8}}$$

$$\bullet\ C_1 = \frac{q_2}{EI}\frac{l}{2} - \left(-\frac{q_2}{EI}\frac{l^{\not{2}}}{8}\right)\frac{1}{\not{l}} \Rightarrow C_1 = \frac{q_2}{EI}\frac{l}{2} + \frac{q_2}{l} + \frac{q_2}{EI}\frac{l}{8} = \frac{4q_2 l + q_2 l}{8EI} = \boxed{\frac{5}{8}\frac{q_2 l}{EI}}$$

VALORI TROVATI

$$C_1 = \frac{5}{8}\frac{ql}{EI} \qquad C_2 = -\frac{ql^2}{8EI} \qquad C_3 = 0 \qquad C_4 = 0$$

VERIFICA AL BORDO

$$v(s) = -\frac{q_2}{EI}\frac{s^4}{24} + C_1\frac{s^3}{6} + C_2\frac{s^2}{2}$$

$v(0) = 0$

$$v(l) = -\frac{q_2}{EI}\cdot\frac{(l)^4}{24} + \left(\frac{5}{8}\frac{ql}{EI}\right)\frac{(l)^3}{6} + \left(-\frac{ql^2}{8EI}\right)\frac{(l)^2}{2} = -\frac{q_2}{EI}\frac{l^4}{24} + \frac{5}{48}\frac{ql^4}{EI} - \frac{1}{16}\frac{l^4}{EI} = \frac{-2q_2 l^4 + 5ql^4 - 3ql^4}{48EI} = 0$$

CONDIZIONI VERIFICATE

SPOSTAMENTO MASSIMO

$$\psi(s) = \frac{dv}{ds} = -\frac{q_2}{EI}\frac{s^3}{6} + C_1\frac{s^2}{2} + C_2 s$$

$$\psi(s) = -\frac{q}{EI}\frac{s^3}{6} + \frac{5}{8}\frac{ql}{EI}\frac{s^2}{2} - \frac{ql^2}{8EI}s = -\frac{qs^3}{6EI} + \frac{5qls^2}{16EI} - \frac{ql^2 s}{8EI}$$

$$-\frac{qs^3}{6EI} + \frac{5qls^2}{16EI} - \frac{ql^2 s}{8EI} = 0 \Rightarrow \frac{qs}{2EI}\left(-\frac{s^2}{3} + \frac{5}{8}sl - \frac{l^2}{4}\right) = 0$$

oppure $\left(\frac{2EI}{l}\right)\frac{qs}{2EI} = 0 \Rightarrow \boxed{s_1 = 0}$ $\qquad -\frac{s^2}{3} + \frac{5sl}{8} - \frac{l^2}{4} = 0$

$$s_{2,3} = \frac{-\frac{5}{8}l \pm \sqrt{\frac{25}{64}l^2 - \frac{1}{3}l^2}}{-\frac{2}{3}} =$$

$$= \frac{-\frac{5}{8}l \pm \sqrt{\frac{25l^2 - 64l^2}{192}}}{-\frac{2}{3}} = \frac{-\frac{5}{8}l \pm \sqrt{\frac{-39l^2}{192}}}{-\frac{2}{3}} =$$

$$= (-0{,}625l \pm 0{,}24l)\left(-\frac{3}{2}\right) = (-0{,}625l \pm 0{,}24l)(-1$$

$$\boxed{s_1 = 1{,}29l} \qquad \boxed{s_2 = 0{,}52l}$$

TROVARE ψ, M, T, R.V.

$$\psi(s) = \frac{q_2}{EI}\frac{s^3}{6} + C_1\frac{s^2}{2} + C_2 s + C_3$$

$$\psi(0) = \frac{q}{EI}\frac{(0)^3}{6} + \frac{5}{8}\frac{ql}{EI}\frac{(0)^2}{2} - \frac{ql^2}{8EI}(0) + 0 = \boxed{0}$$

$$\psi(l) = -\frac{q_2}{EI}\frac{l^3}{6} + \frac{5}{16}\frac{ql^3}{EI} - \frac{ql^3}{8EI} + 0 = \frac{-8ql^3 + 15ql^3 - 6ql^3}{48EI} = \boxed{\frac{1}{48}\frac{ql^3}{EI}}$$

$M(s) = -\frac{q_2}{EI}\frac{s^2}{2} + C_1 s + C_2$ per disegnare l'andamento della funzione del momento.

$$M(0) = -\frac{q}{EI}\frac{(0)^2}{2} + \frac{5}{8}\frac{ql(0)}{EI} - \frac{ql^2}{8EI} = \boxed{-\frac{ql^2}{8EI}}$$

$$M(l) = -\frac{q}{EI}\frac{l^2}{2} + \frac{5}{8}\frac{ql^2}{EI} - \frac{ql^2}{2EI} = \frac{-4ql^2 + 5ql^2 - 1ql^2}{8EI} = \boxed{0}$$ quindi è verificata

Per trovare dove si annulla il momento facciamo la sua derivata prima

$T(s) = \pi'(s) = 0 \Rightarrow -\frac{q s}{EI}\frac{s}{2} + \frac{5}{8}\frac{ql}{EI}\cdot 1 - 0 = 0 \Rightarrow -s = \left(-\frac{5}{8}\frac{l}{EI}\right)\left(\frac{EI}{q}\right) \Rightarrow -s = -\frac{5}{8}l \Rightarrow \boxed{s = \frac{5}{8}l}$

Valori del momento nel punto trovato

$$\pi\left(\frac{5}{8}l\right) = -\frac{1}{2}\frac{ql}{EI}\left(\frac{5}{8}l\right)^2 + \frac{5}{8}\frac{ql}{EI}\left(\frac{5}{8}l\right) - \frac{ql^2}{8EI} = -\frac{25}{128}\frac{ql^2}{EI} + \frac{25}{64}\frac{ql^2}{EI} - \frac{ql^2}{8EI} = \frac{-25ql^2 + 50ql^2 - 16ql^2}{128EI} =$$

$$= \boxed{\frac{9ql^2}{128EI}}$$

EI non viene preso in considerazione perché il momento non dipende dal materiale

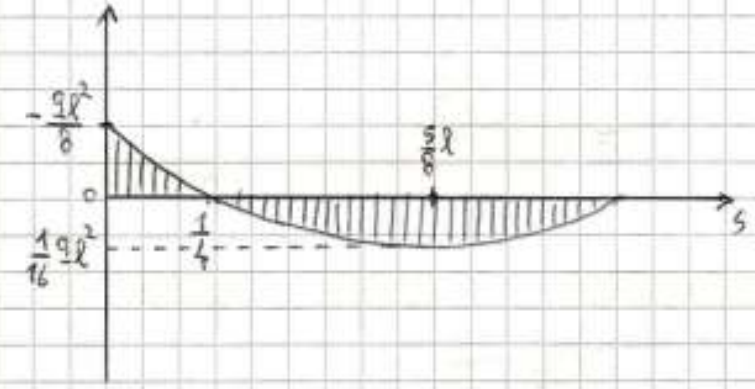
Si pone $\pi(s) = 0$ e si trovano i punti in cui il momento è nullo

$$\pi(s) = \left(-\frac{qs^2}{2} + \frac{5}{8}ql - \frac{ql^2}{8}\right)EI \qquad -\frac{q}{2}s^2 + \frac{5}{8}qls - \frac{ql^2}{8} = 0$$

$$s_{1,2} = \frac{-\frac{5}{8}l \pm \sqrt{\frac{25}{64}l^2 - \frac{1}{4}l^2}}{-1} = \left(-\frac{5}{8}l \pm l\right)\sqrt{\frac{9}{64}}$$

$$s_{1,2} = -\frac{5}{8}l \pm \frac{3}{8}l \quad \begin{cases} l & \boxed{s_1 = l} \\ \frac{1}{4}l & \boxed{s_2 = \frac{1}{4}} \end{cases}$$

DIAGRAMMA DEL MOMENTO

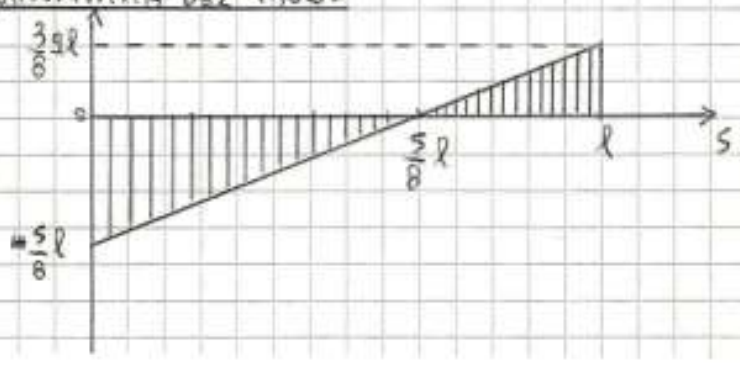


CALCOLIAMO IL TAGLIO

$$T(s) = -\frac{d\pi}{ds} = -\pi'(s) \Rightarrow -\pi'(s) = T(s) = qs - \frac{5}{8}ql$$

$$T(s=0) = \boxed{-\frac{5}{8}ql} \qquad T(s=l) = ql - \frac{5}{8}ql = \boxed{\frac{3}{8}ql}$$

DIAGRAMMA DEL TAGLIO



REAZIONI VINCOLARI

Osservando i valori delle condizioni al bordo del taglio e del momento si trova le reazioni vincolari

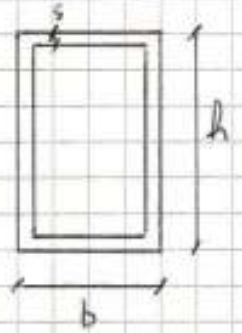
TROVARE IL MASSIMO SPOSTAMENTO

DATI

$q = 20\ kN/m$

$l = 6\ m$

- PROFILO TRAVE



$h = 30\ cm$ $\quad E = 1,999\ MPa$

$b = 20\ cm$ $\quad I = 120\,200\,000\ mm^4$

$s = 1\ cm$

$$v(0,52l) = \frac{-q}{24EI}(0,52l)^2 + \frac{5ql}{48EI}(0,52l)^3 - \frac{1}{16}\frac{q_2}{EI}(0,52l)^2$$

$0,52 \cdot 6 = 3,42\ m$

$$v(3,42) = \frac{-20(3420)^4}{24EI} + \frac{5 \cdot 20 \cdot 6000 (3420)^3}{48EI} - \frac{20(6000)^2 \cdot (3420)^2}{16EI}$$

$v_{(s)} = -4,225 + 20,223 - 21,814 = 5,816\ mm$

$v(s) = \boxed{-5,816\ m}$

DIAGRAMMI CON VALORI

DIAGRAMMA MOMENTO

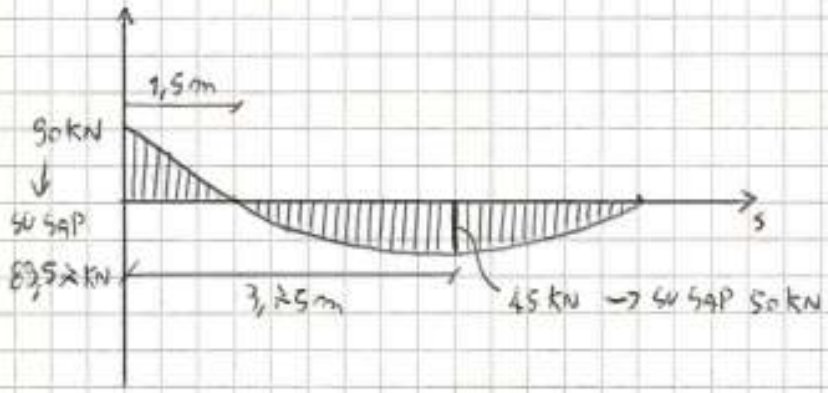
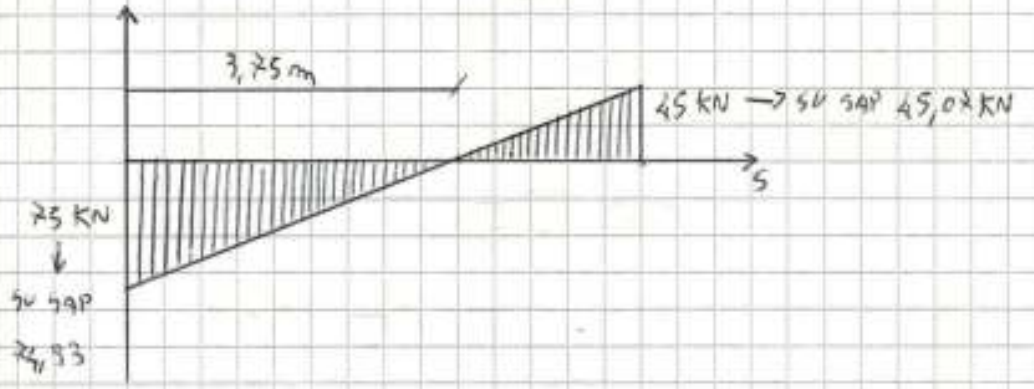
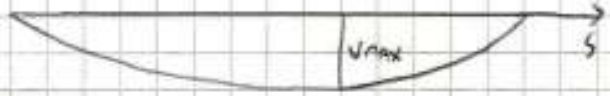


DIAGRAMMA DEL TAGLIO



SPOSTAMENTO MASSIMO



$V_{max} = -0,0058\ m \rightarrow$ SU SAP $-0,0055\ mm$