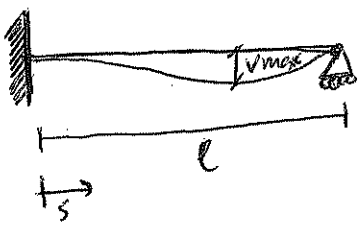
Esercitazione Linea elastica



eq. di bilancio | conguenza | legame costitutivo

$$\begin{cases} N' + q_1 = 0 \\ T' + q_2 = 0 \\ M' + T = 0 \end{cases} \quad \begin{cases} \varepsilon = u' \\ v' = \varphi \\ \chi = \varphi' \end{cases} \quad \begin{cases} N = EA\varepsilon \\ M = EI\chi \end{cases}$$

$$\begin{cases} \frac{dN}{ds} + q_1 = 0 \\ N = EA\varepsilon \\ \varepsilon = \frac{du}{ds} \end{cases} \quad \begin{cases} \frac{dT}{ds} + q_2 = 0 \\ \frac{dM}{ds} + T = 0 \\ M = EI\chi \\ \chi = \frac{d\varphi}{ds} \\ \varphi = \frac{dv}{ds} \end{cases}$$

$$\frac{dT}{ds} + q_2 = 0 \longrightarrow T = -\frac{dM}{ds}$$

$$\frac{d}{ds}\left(-\frac{dM}{ds}\right) + q_2 = 0$$

$$-\frac{d^2M}{ds} + q_2 = 0$$

sostituisco φ e χ

$$\chi = \frac{d^2v}{ds^2} \quad \varphi = \frac{dv}{ds} \longrightarrow \chi = \frac{d\varphi}{ds} = \frac{d}{ds}\left(\frac{dv}{ds}\right) = \frac{d^2v}{ds^2}$$

$$\boxed{M = EI\chi = EI\frac{d^2v}{ds^2}}$$

sapendo che

$$\frac{d^2M}{ds^2} = q_2$$

$$\frac{d^2}{ds^2}\left(EI\frac{d^2v}{ds^2}\right) = q_2$$

$$\boxed{EI\frac{d^4v}{ds^4} = q_2}$$

$$\frac{EI}{EI}\frac{d^4v}{ds^4} = \frac{q_2}{EI}$$

integriamo per ricavare $v(s)$:

$$\frac{d^4v}{ds^4} = \frac{q_2}{EI}$$

$$\frac{d^3v}{ds^3} = \frac{q_2}{EI}s + C_1$$

$$\frac{d^2v}{ds^2} = \frac{q_2}{EI}\frac{s^2}{2} + C_1 s + C_2$$

$$\frac{dv}{ds} = \frac{q_2}{EI}\frac{s^3}{6} + C_1\frac{s^2}{2} + C_2 s + C_3$$

$$\boxed{v(s) = \frac{q_2}{EI}\frac{s^4}{24} + C_1\frac{s^3}{6} + C_2\frac{s^2}{2} + C_3 s + C_4}$$