TRAVE IPERSTATICA (svolta con il metodo della linea elastica)

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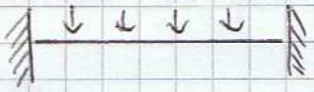


Condizioni al bordo

$M(0) \neq 0$ $\quad \varphi(0) = 0$ $\quad M(l) \neq 0$ $\quad \varphi(l) = 0$

$T(0) \neq 0$ $\quad V(0) = 0$ $\quad T(l) \neq 0$ $\quad V(l) = 0$

$$V(s) = -\frac{q_z s^4}{EJ\,24} + C_1 \frac{s^3}{6} + C_2 \frac{s^2}{2} + C_3 s + C_4$$

$$\varphi(s) = -\frac{q_z s^3}{EJ\,6} + C_1 \frac{s^2}{2} + C_2 s + C_3$$

$$M(s) = -EJ\left(\frac{q_z s^2}{EJ\,2} + C_1 s + C_2\right)$$

$$T(s) = -EJ\left(\frac{q_z s}{EJ} + C_1\right)$$

$s = 0$

$V(0) = C_4 \rightarrow \boxed{C_4 = 0}$

$\varphi(0) = C_3 \rightarrow \boxed{C_3 = 0}$

$s = l$

$$\begin{cases} V(s) = -\dfrac{q_z l^4}{EJ\,24} + C_1 \dfrac{l^3}{6} + C_2 \dfrac{l^2}{2} = 0 \quad * \\ \varphi(s) = -\dfrac{q_z l^3}{EJ\,6} + C_1 \cdot \dfrac{l^2}{2} + C_2 \cdot l \end{cases}$$

$$\frac{1}{\cancel{l}} C_2 \cdot \cancel{l} = \left(-\frac{q_z l^{\cancel{3}\,2}}{EJ\,6} - C_1 \cdot \frac{l^{\cancel{3}}}{2}\right) \frac{1}{\cancel{l}}$$

$$C_2 = -\frac{q_z l^2}{EJ\,6} - C_1 \frac{l}{2}$$

$$* = -\frac{q_z l^4}{EJ\,24} + C_1 \frac{l^3}{6} + \frac{l^2}{2}\left(-\frac{q_z l^2}{EJ\,6} - C_1 \frac{l}{2}\right) =$$

$$= -\frac{q_z l^4}{EJ\,24} + C_1 \frac{l^3}{6} + \frac{q_z l^2}{2EJ}\left(\frac{l^2}{6}\right) - C_1 \frac{l^3}{4} = \&$$

$$= -\frac{q_2 l^4}{EJ\,24} + C_1 \cdot \frac{l^3}{6} + \frac{q_2 l^4}{12EJ} - C_1 \frac{l^3}{4} = -\frac{q_2 l^4}{\cancel{24}\,2} - C_1 \frac{\cancel{l^3}}{\cancel{12}} \cdot \frac{\cancel{12}}{\cancel{l^3}}$$

$$\boxed{C_1 = \frac{1}{2}\frac{q_2 l}{EJ}}$$

$$C_2 = -\frac{q_2 l^2}{EJ\,6} - C_1 \frac{l}{2}$$

$$C_2 = -\frac{q_2 l^2}{EJ\,6} + \frac{q_2 l}{EJ}\left(\frac{l}{2}\right)$$

$$C_2 = -\frac{q_2 l^2}{EJ\,6} + \frac{q_2 l^2}{4EJ}$$

$$C_2 = -\frac{q_2 l^2}{6EJ} + \frac{q_2 l^2}{4EJ}$$

$$\boxed{C_2 = \frac{q_2 l^2}{12EJ}}$$

Sostituisco $s=0$

$$V(0) = \frac{q_2 s^4}{EJ\,24} - \frac{q_2 l}{2EJ}\left(\frac{s^3}{6}\right) + \frac{q_2 l^2}{12EJ}\left(\frac{s^2}{2}\right)$$

$V(0) = 0$ VERIFICATA

$$\varphi(0) = \frac{q_2 s^3}{EJ\,6} - \frac{q_2 l}{2EJ} + \frac{q_2 l^2}{12EJ} \cdot s$$

$\varphi(0) = 0$ VERIFICATA

Sostituisco $s = l$

$$V(l) = -\frac{q_2 l^4}{EJ\,24} + \frac{q_2 l}{EJ} \cdot \frac{l^3}{6} - \frac{q_2 l^2}{12EJ}\frac{l^2}{2}$$

$$-\frac{q_2 l^4}{24EJ} + \frac{q_2 l^4}{12EJ} - \frac{q_2 l^4}{24EJ} = \frac{-1+2-1}{24} = 0$$

$V(l) = 0$ ✓

$$\varphi(l) = -\frac{q_2}{EJ} \cdot \frac{l^3}{6} + \frac{1}{2}\frac{q_2 l}{EJ}\frac{l^2}{2} - \frac{q_2 l^2}{12EJ} \cdot l$$

$$-\frac{q_2 l^3}{6EJ} + \frac{q_2 l^3}{4EJ} - \frac{q_2 l^3}{12EJ} = 0 \qquad \frac{-2+3-1}{12} = 0 \quad \varphi(l) = 0 ✓$$

Per trovare i punti in cui la deformata assume valori di massimo e di minimo bisogna porre $\varphi(s)=0$

$$\varphi(s)=0 \qquad \frac{12EJ}{q_2}\left(\frac{q_2 s^3}{6EJ} - \frac{q_2\ell}{4EJ}s^2 + \frac{q_2\ell^2 s}{12EJ}\right)=0$$

$$2s^3 - 3s^2\ell + \ell^2 s = 0$$

$$s(2s^2 - 3s\ell + \ell^2) = 0$$

$$s_1 = 0$$

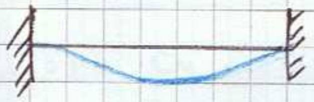
$$2s^2 - 3s\ell + \ell^2 = 0$$

$$s_{2,3} = 3\ell \pm \frac{\sqrt{9\ell^2 - 4(2)(\ell^2)}}{4} = \frac{3\ell \pm \sqrt{\ell^2}}{4} = \frac{3\ell \pm \ell}{4}$$

$$s_2 = 1\ell$$

$$s_3 = \frac{1}{2}\ell$$

I punti di massimo e minimo sono $s_1 = 0$; $s_2 = \frac{1}{2}$; $s_3 = \ell$



Calcoliamo taglio e momento:

$$M(o) = EJ\left(-\frac{q_2}{EJ}\frac{s^2}{2} + C_1 \cdot s + C_2\right)$$

$$M(0) = +C_2 = -\frac{q_2\ell^2}{12}$$

$$M(\ell) = -EJ\left(\frac{q_2\ell}{2EJ} - \frac{q_2\ell^2}{2EJ} + \frac{q_2\ell^2}{12EJ}\right)$$

$$M(\ell) = -\frac{q_2\ell^2}{12}$$

$$T(o) = -EJ\left(-\frac{q_2 s}{EJ} + C_1\right)$$

$$T(o) = C_1 = -\frac{q_2\ell}{2}$$

$$T(\ell) = -EJ\left(-\frac{q_2\ell}{EJ} - \frac{q_2\ell}{2EJ}\right) = +\frac{1}{2}q\ell$$

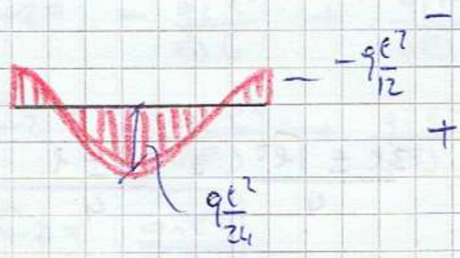
$$M\left(\frac{\ell}{2}\right) = q_2\frac{\ell^2}{24}$$

MOMENTO MAX TAGLIO ZERO.

$$T\left(\frac{\ell}{2}\right) = 0$$

TAGLIO

MOMENTO



REAZIONI VINCOLARI