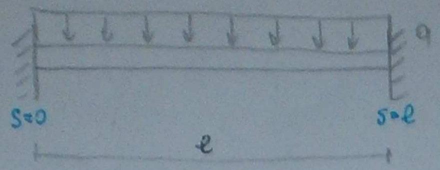
STRUTTURA IPERSTATICA



CONDIZIONI AL CONTORNO

$v(0)=0$ $\qquad$ $v(l)=0$

$\frac{dv}{ds}(0)=0$ $\qquad$ $\frac{dv}{ds}(l)=0$

Con il metodo degli spostamenti o equazioni della linea elastica consente di determinare le reazioni vincolari, i diagrammi delle sollecitazioni, gli spostamenti e deformazioni a partire dalle equazioni di bilancio, equazioni di congruenza e legame costitutivo.
Avremmo:

$$\begin{cases} \frac{dN}{ds}+q_1=0 \\ \varepsilon=\frac{du}{ds} \\ N=EA\cdot\varepsilon \end{cases}$$

Problema flessionale

$$\begin{cases} \frac{dT}{ds}+q_2=0 \\ \frac{dM}{ds}+T=0 \\ M=EJ\cdot\chi \\ \chi=\frac{d\varphi}{ds} \\ \varphi=\frac{dv}{ds} \end{cases}$$

Si ottiene che la curvatura è uguale alla derivata seconda dello spostamento

$$\chi=\frac{d}{ds}\left(\frac{dv}{ds}\right)\rightarrow \boxed{\chi=\frac{d^2v}{ds^2}} \longrightarrow \boxed{M=EJ\,\frac{d^2v}{ds^2}}$$

$$v(\ell) = -\frac{q_2 \ell^4}{24EJ} + \frac{c_1 \ell^3}{6} + \frac{c_2 \ell^2}{2} = 0$$

$$\frac{dv}{ds}(\ell) = -\frac{q_2 \ell^3}{6EJ} + \frac{c_1 \ell^2}{2} + c_2 \ell = 0$$

$$c_2 \ell = \frac{q_2 \ell^3}{6EJ} - \frac{c_1 \ell^2}{2}$$

$$c_2 = \frac{q_2 \ell^2}{6EJ} - \frac{c_1 \ell}{2} \rightarrow c_2 = \frac{q_2 \ell^2}{6EJ} - \frac{q_2 \ell^2}{4EJ} \qquad \boxed{c_2 = -\frac{q_2 \ell^2}{12EJ}}$$

$$-\frac{q_2 \ell^4}{24EJ} + \frac{c_1 \ell^3}{6} + \left(+\frac{q \ell^2}{6EJ} - \frac{c_1 \ell}{2}\right) \cdot \frac{\ell^2}{2} = 0$$

$$-\frac{q_2 \ell^4}{24EJ} + \frac{c_1 \ell^3}{6} + \frac{q \ell^4}{12EJ} - \frac{c_1 \ell^3}{4} = 0$$

$$\frac{q_2 \ell^4}{24EJ} - \frac{c_1 \ell^3}{12} = 0$$

$$\frac{c_1 \ell^3}{12} = \frac{q_2 \ell^4}{24EJ} \qquad c_1 = \left(\frac{q_2 \ell^4}{24EJ}\right) \cdot \frac{12}{\ell^3} = \qquad \boxed{c_1 = \frac{q_2 \ell}{2EJ}}$$

$v(0) = 0$

$$v(\ell) = -\frac{q_2 \ell^4}{24EJ} + \frac{q_2 \ell^4}{12EJ} - \frac{q_2 \ell^4}{24EJ} = 0$$

Equazione della rotazione

$$\varphi(s) = -\frac{q_2 s^3}{EJ6} + \frac{q_2 \ell}{2EJ}\frac{s^2}{2} + \frac{q_2 \ell^2}{12EJ} \cdot s$$

$$\frac{q_2 s}{12EJ}\left(-2s^2 + 3\ell s + \ell^2\right) = 0$$

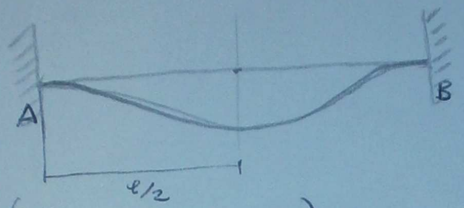
$$\frac{q_2 s}{12EJ} = 0 \qquad \boxed{s_1 = 0}$$

$$\frac{3\ell \pm \sqrt{9\ell^2 - 8\ell^2}}{2} \qquad \boxed{s_2 = \ell} \qquad \boxed{s_3 = \frac{\ell}{2}}$$

$$\frac{3\ell \pm \sqrt{9\ell^2 - 8\ell^2}}{4} = \frac{3\ell + \ell}{4} \quad \begin{cases} \frac{4\ell}{4} = \boxed{\ell} \\ \frac{2\ell}{4} = \boxed{\frac{\ell}{2}} \end{cases} \qquad \frac{q_2}{12EJ}s = 0 \quad \boxed{s_3 = 0}$$ ③

$s_3 = 0 \quad s_2 = \frac{\ell}{2} \quad s_1 = \ell$

I punti di massimo della funzione spostamento si trovano uno in 0, $\frac{\ell}{2}$ e ℓ ed avrà valore $v_{max} = v(\frac{\ell}{2})$ $v_{min} = v(0)$ $v_{min} = v(\ell)$



$$M(s) = -EJ \frac{d^2v}{ds} = -EJ\left(\frac{q_2 s^2}{2EJ} + C_1 s + C_2\right)$$

$$= -EJ\left(\frac{q_2 s^2}{2EJ} - \frac{q_2 \ell}{2EJ} s + \frac{q_2 \ell^2}{12EJ}\right)$$

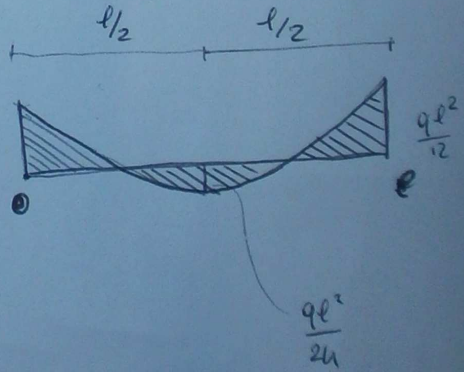
$$M(s) = -\frac{q_2 s^2}{2} + \frac{q_2 \ell s}{2} - \frac{q_2 \ell^2}{12}$$

$$\boxed{M(0) = -\frac{q_2 \ell^2}{12}}$$

$$M\left(\frac{\ell}{2}\right) = -\frac{q_2}{2} \cdot \frac{\ell^2}{4} + \frac{q_2 \ell}{2} \cdot \frac{\ell}{2} - \frac{q_2 \ell^2}{12}$$

$$-\frac{q_2 \ell^2}{8} + \frac{q_2 \ell^2}{4} - \frac{q_2 \ell^2}{12} = -\frac{q_2 \ell^2}{24}$$

$$\boxed{M\left(\frac{\ell}{2}\right) = -\frac{q \ell^2}{24}}$$



$$M(\ell) = -\cancel{\frac{q_2 \ell^2}{2}} + \cancel{\frac{q_2 \ell^2}{2}} - \frac{q_2 \ell^2}{12}$$

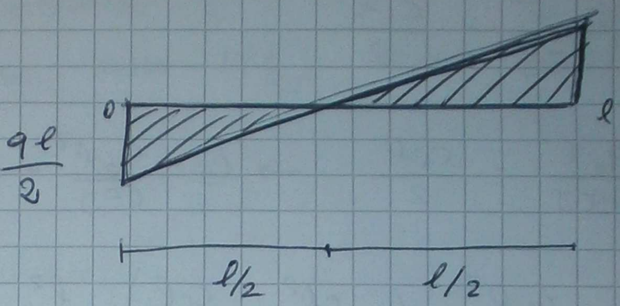
$$\boxed{M(\ell) = -\frac{q \ell^2}{12}}$$

~~Caratteristiche~~ sforzi

$$T = \frac{dM(s)}{ds} = EJ\left(\frac{q_2}{EJ}s + c_1\right)$$

$$T(s) = -EJ\left(-\frac{q_2}{EJ}s + \frac{q_2 l}{2EJ}\right)$$

$$T(s) = \frac{q_2}{\cancel{EJ}} - \frac{q_2 l}{2}$$



$$T(0) = -\frac{q_2 l}{2}$$

$$Tl = \cancel{\frac{l}{EJ}}\, q_2 l - \frac{q_2 l}{2} = \frac{q_2 l}{2}$$

$$T\left(\frac{l}{2}\right) = q_2\frac{l}{2} - q_2\frac{l}{2} = 0$$

CALCOLIAMO LE REAZIONI VINCOLARI