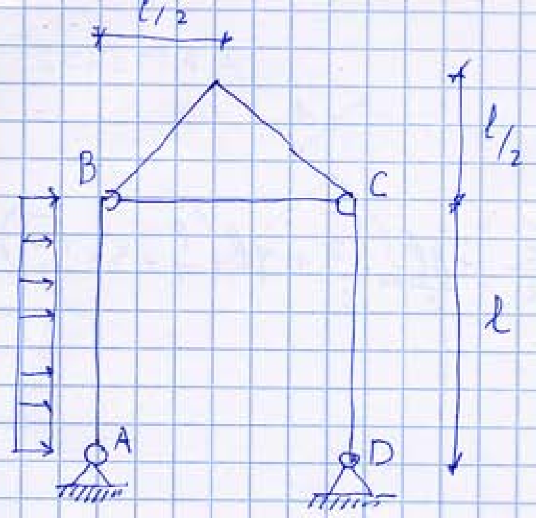
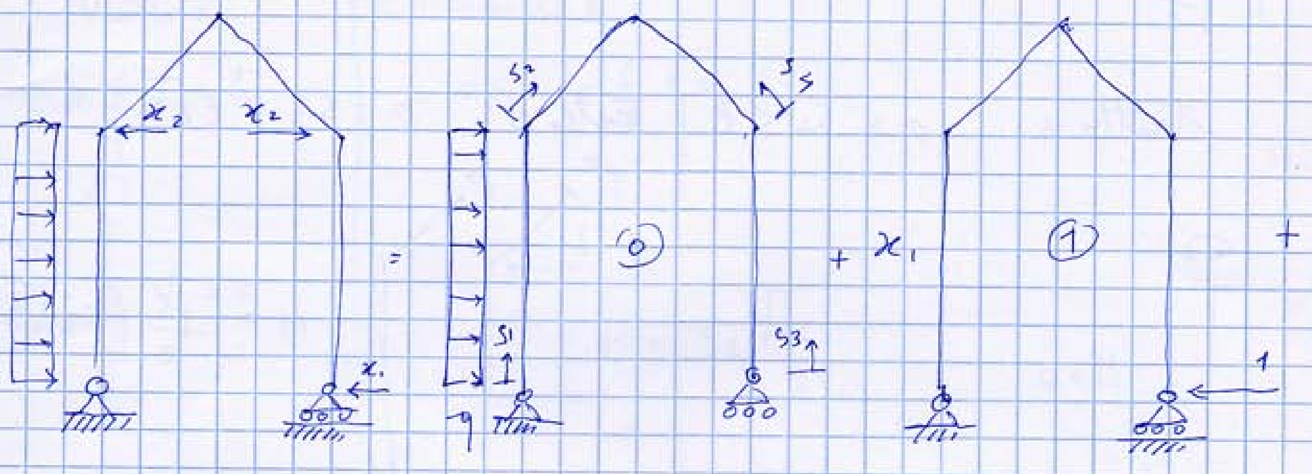
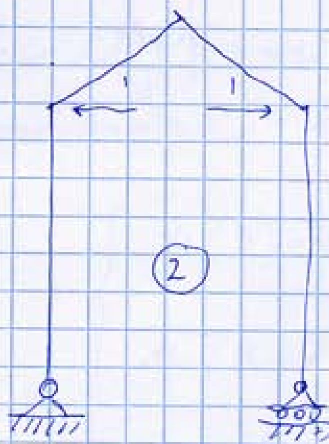
VALLICELLI



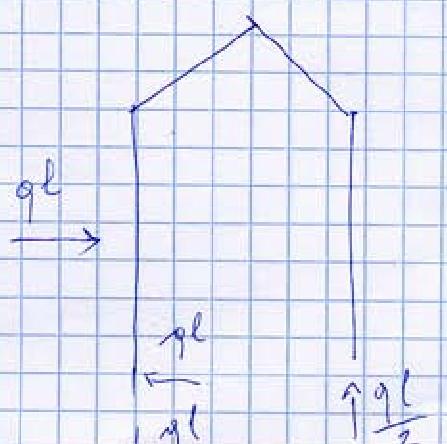
struttura 2 volte iperstatica



$+ x_2$

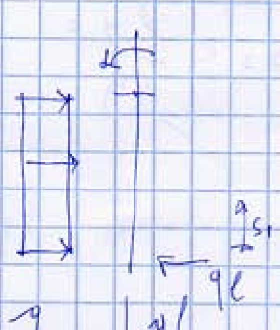


SCHEMA 0



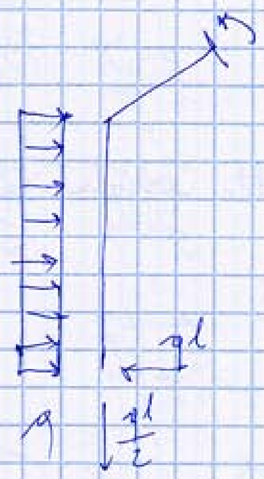
tretto 1

$0 \le s_1 < l$



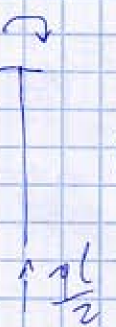
$$\hat{M} = -qls_1 + \frac{qs_1^2}{2}$$

tratto 2 $\quad 0 \leq s_2 < \frac{l\sqrt{2}}{2}$



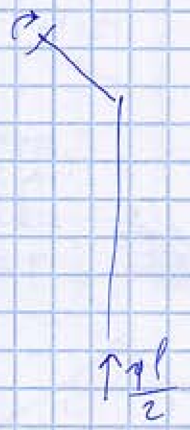
$$M = -ql\left(l + s_2\frac{\sqrt{2}}{2}\right) + \frac{ql}{2}s_2\frac{\sqrt{2}}{2} + ql\left(\frac{l}{2} + s_2\frac{\sqrt{2}}{2}\right)$$

tratto 4 $\quad 0 \leq s_4 \leq l$



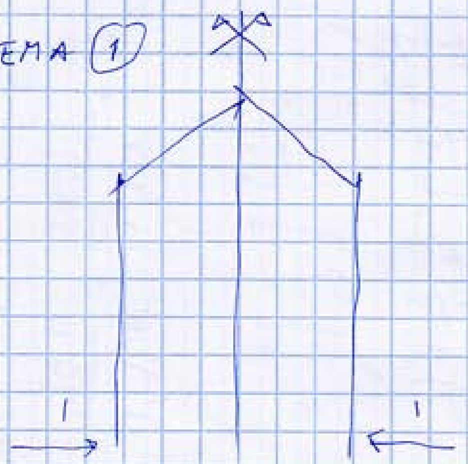
$M = 0$

tratto 3 $\quad 0 \leq s_3 < l\sqrt{2}$



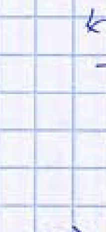
$$M = -\frac{ql}{2}\left(s_3\frac{\sqrt{2}}{2}\right)$$

SCHEMA 1



tratto 1

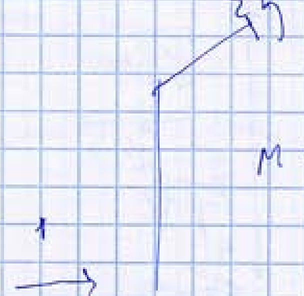
$0 \leq s_1 < l$



$M = s_1$

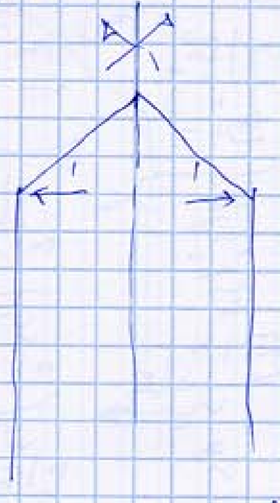
tratto 2

$0 \leq s_2 < \frac{l\sqrt{2}}{2}$



$M = s_2\frac{\sqrt{2}}{2}$

SCHEMA 2



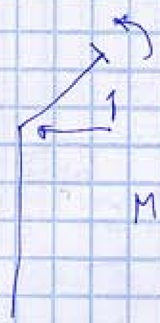
tratto 1

$0 \le s_1 < l$

$M = 0$

tratto 2

$0 \le s_1 < \frac{l}{2}\sqrt{2}$



$M = s_2 \frac{\sqrt{2}}{2}$

applico il principio dei lavori virtuali per individuare le due reazioni iperstatiche.

$$\begin{cases} u(D) = 0 & (1) \\ u(B) = u(C) \Longleftrightarrow u(B) - u(C) = 0 & (2) \end{cases}$$

$$1 \cdot u(D) = \frac{1}{EJ}\int_L M^0 M^1 ds + \frac{x_1}{EJ}\int_L {M^1}^2 ds + \frac{x_2}{EJ}\int_L M^1 M^2 ds = 0$$

$$0=\frac{1}{EI}\int_0^{l} s_1\left(\frac{qs_1^2}{2}-qls_1\right)ds_1+\frac{x_1}{EI}\int_0^{l} s_1^2\,ds_1+\frac{1}{EI}\int_0^{\frac{l\sqrt2}{2}}\left[-ql\left(l+s_2\frac{\sqrt2}{2}\right)+\frac{qls_2\sqrt2}{4}+ql\left(\frac{l}{2}+s_2\frac{\sqrt2}{2}\right)\right]s_2\frac{\sqrt2}{2}\,ds_2+\frac{x_1}{EI}\int_0^{\frac{l\sqrt2}{2}}\left(s_2\frac{\sqrt2}{2}\right)^2ds_2+\frac{x_2}{EI}\int_0^{\frac{l\sqrt2}{2}}\left(s_2\frac{\sqrt2}{2}\right)\left(-s_2\frac{\sqrt2}{2}\right)$$

$$+\frac{1}{EI}\int_0^{\frac{l\sqrt2}{2}}-\frac{ql}{2}\left(s_3\frac{\sqrt2}{2}\right)s_3\frac{\sqrt2}{2}\,ds_3+\frac{x_1}{EI}\int_0^{\frac{l\sqrt2}{2}}\left(s_3\frac{\sqrt2}{2}\right)^2ds_3+\frac{x_2}{EI}\int_0^{\frac{l\sqrt2}{2}}\left(s_3\frac{\sqrt2}{2}\right)\left(-s_3\frac{\sqrt2}{2}\right)ds_3+\frac{x_1}{EI}\int_0^{l}s_4^2\,ds_4=$$

$$=\frac{1}{EI}\int_0^{l}\frac{qs_1^3}{2}-\frac{1}{EI}\int_0^{l}qls_1^2\,ds_1+\frac{x_1}{EI}\int_0^{l}s_1^2\,ds_1+\frac{1}{EI}\int_0^{\frac{l\sqrt2}{2}}-ql^2s_2\frac{\sqrt2}{2}\,ds_2+\frac{1}{EI}\int_0^{\frac{l\sqrt2}{2}}-qls_2^2\frac{1}{2}\,ds+\frac{1}{EI}\int_0^{\frac{l\sqrt2}{2}}\frac{qls_2^2}{4}\,ds_2+\frac{1}{EI}\int_0^{\frac{l\sqrt2}{2}}\frac{ql^2s_2}{2}\,ds_2+\frac{1}{EI}\int_0^{\frac{l\sqrt2}{2}}\frac{qls_2^2}{2}\,ds_2+$$

$$+\frac{x_1}{EI}\int_0^{\frac{l\sqrt2}{2}}\frac{s_2^2}{2}\,ds_2+\frac{x_2}{EI}\int_0^{\frac{l\sqrt2}{2}}-\frac{s_2^2}{2}\,ds_2+\frac{1}{EI}\int_0^{\frac{l\sqrt2}{2}}-\frac{qls_3^2}{4}\,ds_3+\frac{x_1}{EI}\int_0^{\frac{l\sqrt2}{2}}\frac{s_3^2}{2}\,ds_3+\frac{x_2}{EI}\int_0^{\frac{l\sqrt2}{2}}-\frac{s_3^2}{2}\,ds_3+\frac{x_1}{EI}\int_0^{l}s_4^2\,ds_4=$$

$$=\frac{q}{2EI}\left[\frac{s^4}{4}\right]_0^l-\frac{ql}{EI}\left[\frac{s^3}{3}\right]_0^l+\frac{x_1}{EI}\left[\frac{s^3}{3}\right]_0^l-\frac{ql^2\sqrt2}{2EI}\left[\frac{s^2}{2}\right]_0^{\frac{l\sqrt2}{2}}-\frac{ql}{2EI}\left[\frac{s^3}{3}\right]_0^{\frac{l\sqrt2}{2}}+\frac{ql^2}{4EI}\left[\frac{s^2}{2}\right]_0^{\frac{l\sqrt2}{2}}+\frac{ql}{2EI}\left[\frac{s^3}{3}\right]_0^{\frac{l\sqrt2}{2}}+\frac{x_1}{2EI}\left[\frac{s^3}{3}\right]_0^{\frac{l\sqrt2}{2}}$$

$$-\frac{x_2}{2EI}\left[\frac{s^3}{3}\right]_0^{\frac{l\sqrt2}{2}}-\frac{ql}{4EI}\left[\frac{s^3}{3}\right]_0^{\frac{l\sqrt2}{2}}+\frac{x_1}{2EI}\left[\frac{s^3}{3}\right]_0^{\frac{l\sqrt2}{2}}-\frac{x_2}{2EI}\left[\frac{s^3}{3}\right]_0^{\frac{l\sqrt2}{2}}+\frac{x_1}{EI}\left[\frac{s^3}{3}\right]_0^l=$$

$$=\frac{ql^4}{8EI}-\frac{ql^4}{3EI}+\frac{2x_1l^3}{3EI}-\frac{ql^4\sqrt2}{8EI}-\cancel{\frac{ql^4 2\sqrt2}{24EI}}+\cancel{\frac{ql^4\sqrt2}{48EI}}+\cancel{\frac{ql^4}{8EI}}+\cancel{\frac{ql^4\sqrt2}{24EI}}+\frac{x_1l^3\sqrt2}{12EI}-\frac{x_2l^3\sqrt2}{12EI}-\cancel{\frac{ql^4\sqrt2}{48EI}}+\cancel{\frac{x_1l^3\sqrt2}{24EI}}+$$

$$-\cancel{\frac{x_2l^3\sqrt2}{24EI}}+\cancel{\frac{x_1l^3}{3EI}}=x_1\left(\frac{2l^3}{3}+\frac{l^3\sqrt2}{12}\right)-x_2\cdot\frac{l^3\sqrt2}{12}+ql^4\left(\frac{1}{4}-\frac{1}{3}-\frac{\sqrt2}{8}\right)=x_1\left[\frac{(8+\sqrt2)l}{\cancel{12}}\right]-x_2\left[\frac{l\sqrt2}{\cancel{12}}\right]-ql^2\left[\frac{2\sqrt2}{24EI}\right]=$$

$$=0$$

VALLICELLI

$$\mu(B)=\mu(C) \Longleftrightarrow -\mu(B)\cdot 1=\mu(C)\cdot 1 \Longleftrightarrow [\mu(B)+\mu(A)]\cdot 1=0 \Longleftrightarrow \mu(B)+\mu(A)=0$$

$$\frac{1}{EJ}\int_0^{l\sqrt{2}} q l s_2^2 \frac{\sqrt{2}}{2}\, ds_2 + \frac{1}{EJ}\int_0^{\frac{l\sqrt{2}}{2}} \frac{q l s_2^2}{2}\, ds_2 - \frac{1}{EJ}\int_0^{\frac{l\sqrt{2}}{2}} \frac{q l s_2^2}{4}\, ds_2 - \frac{1}{EJ}\int_0^{\frac{l\sqrt{2}}{2}} \frac{q l^2 s_2 \sqrt{2}}{4} - \frac{1}{EJ}\int_0^{\frac{l\sqrt{2}}{2}} \frac{q l s_2^2\, ds_2}{2} - \frac{x_1}{EJ}\int_0^{\frac{l\sqrt{2}}{2}} \frac{s_2^2}{2}\, ds_2 + \frac{x_2}{EJ}\int_0^{\frac{l\sqrt{2}}{2}} \frac{s_2^2}{2}\, ds_2 +$$

$$+\frac{1}{EJ}\int_0^{\frac{l\sqrt{2}}{2}} \frac{q l s_3^2}{4}\cdot ds_3 - \frac{x_1}{EJ}\int_0^{\frac{l\sqrt{2}}{2}} \frac{s_3^2}{2}\, ds_3 + \frac{x_2}{EJ}\int_0^{\frac{l\sqrt{2}}{2}} \frac{s_3^2}{2}\, ds_3 =$$

$$= \frac{q l^2\sqrt{2}}{2EJ}\left[\frac{s^2}{2}\right]_0^{\frac{l\sqrt{2}}{2}} + \frac{q l}{2EJ}\left[\frac{s^3}{3}\right]_0^{\frac{l\sqrt{2}}{2}} - \frac{q l}{4EJ}\left[\frac{s^3}{3}\right]_0^{\frac{l\sqrt{2}}{2}} - \frac{q l^2\sqrt{2}}{4EJ}\left[\frac{s^2}{2}\right]_0^{\frac{l\sqrt{2}}{2}} - \frac{q l}{2EJ}\left[\frac{s^3}{3}\right]_0^{\frac{l\sqrt{2}}{2}} - \frac{x_1}{2EJ}\left[\frac{s^3}{3}\right]_0^{\frac{l\sqrt{2}}{2}} + \frac{x_2}{2EJ}\left[\frac{s^3}{3}\right]_0^{\frac{l\sqrt{2}}{2}} + \frac{q l}{4EJ}\left[\frac{s^3}{3}\right]_0^{\frac{l\sqrt{2}}{2}} +$$

$$-\frac{x_1}{2EJ}\left[\frac{s^3}{3}\right]_0^{\frac{l\sqrt{2}}{2}} + \frac{x_2}{2EJ}\left[\frac{s^3}{3}\right]_0^{\frac{l\sqrt{2}}{2}} =$$

$$= \cancel{\frac{q l^4\sqrt{2}}{8}} + \cancel{\frac{q l^4\sqrt{2}}{24}} - \cancel{\frac{q l^4\sqrt{2}}{48}} + \frac{q l^4\sqrt{2}}{16} - \cancel{\frac{q l^4\sqrt{2}}{24}} - \frac{x_1 l^3\sqrt{3}}{12} + \frac{x_2 l^3\sqrt{2}}{12} + \cancel{\frac{q l^4\sqrt{2}}{48}} - \cancel{\frac{x_1 l^3\sqrt{2}}{24}} + \cancel{\frac{x_2 l^3\sqrt{2}}{24}} =$$

$$= -x_1\left[\frac{l^3\sqrt{2}}{12_3}\right] + x_2\left[\frac{l^3\sqrt{2}}{12_{6_3}}\right] + \frac{q l^4\sqrt{2}}{16_{8_4}} = 0$$

$$-x_1\frac{l}{3} + x_2\frac{l}{3} + \frac{q l^2}{4} = 0$$

$$\begin{cases} x_1(8+\sqrt{2})l - x_2 l\sqrt{2} - \dfrac{ql^2(2+\sqrt{2})}{2} = 0 \\ -x_1\dfrac{l}{3} + x_2\dfrac{l}{3} + \dfrac{ql^2}{4} = 0 \end{cases}$$

$$x_1 = \frac{3\left(\frac{x_2 l}{3} + \frac{ql^2}{4}\right)}{l} = \frac{\cancel{3}\cancel{l}(4x_2+3ql)}{4\cdot\cancel{3}\cancel{l}} = \frac{4x_2+3ql}{4}$$

$$l\frac{(4x_2+3ql)}{4}(8+\sqrt{2}) - x_2 l\sqrt{2} - \frac{ql^2(2+\sqrt{2})}{2} = 0$$

$$l(4x_2+3ql)(8+\sqrt{2}) - 4x_2 l\sqrt{2} - 2ql^2(2+\sqrt{2}) = 0$$

$$(4x_2 l + 3ql^2)(8+\sqrt{2}) - 4x_2 l\sqrt{2} - 4ql^2 - 2\sqrt{2}ql^2 = 0$$

$$32x_2 \cancel{l} + \cancel{4\sqrt{2}x_2 l} + 24ql^{\cancel{2}} + 3\sqrt{2}ql^{\cancel{2}} - \cancel{4\sqrt{2}l x_2} - 4ql^{\cancel{2}} - 2\sqrt{2}ql^{\cancel{2}} = 0$$

$$x_2 = \frac{(24+3\sqrt{2}-4-2\sqrt{2})}{32}ql = \boxed{\frac{20+\sqrt{2}}{32}ql}$$

$$x_1 = \frac{\left(\frac{20+\sqrt{2}}{8}+3\right)ql}{4} = \frac{44+\sqrt{2}}{8}\cdot\frac{1}{4}ql = \boxed{\frac{44+\sqrt{2}}{32}ql}$$