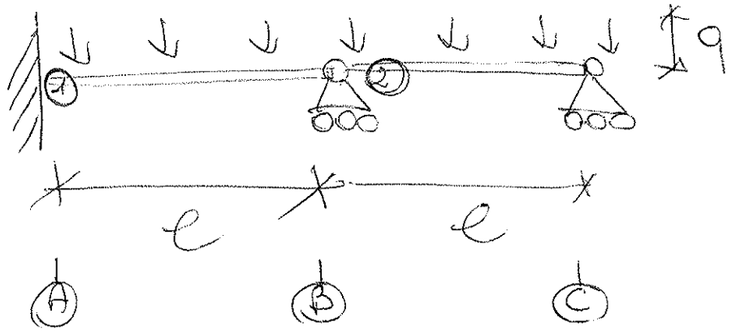
ESERCITAZIONE 16/03/2011



CONDIZIONI CINEMATICA / STATICA

$V_1(0)=0$ $\quad M_1(\ell)=M_2(0)$ $\quad V_2(\ell)=0$

$\varphi_1(0)=0$ $\quad \varphi_1(\ell)=\varphi_2(0)$ $\quad M_2(\ell)=0$

$V_1(\ell)=0$

$V_2(0)=0$

TRATTO 1

$$V_1(s) = \frac{-q}{EJ}\cdot\frac{s^4}{24} + C_1\frac{s^3}{6} + C_2\frac{s^2}{2} + C_3 s + C_4$$

$$\varphi_1(s) = \frac{-q}{EJ}\cdot\frac{s^3}{6} + C_1\frac{s^2}{2} + C_2 s + C_3$$

$$M_1(s) = \frac{-q}{EJ}\cdot\frac{s^2}{2} + C_1 s + C_2$$

TRATTO 2

$$V_2(s) = \frac{-q}{EJ}\cdot\frac{s^4}{24} + C_5\frac{s^3}{6} + C_6\frac{s^2}{2} + C_7 s + C_8$$

$$\varphi_2(s) = \frac{-q}{EJ}\cdot\frac{s^3}{6} + C_5\frac{s^2}{2} + C_6 s + C_7$$

$$M_2(s) = \frac{-q}{EJ}\cdot\frac{s^2}{2} + C_5 s + C_6$$

VERIFICA

$V_1(0)=0$ ✓ $\qquad \varphi_1(0)=0$ ✓

$V_2(\ell)=0 \qquad -\frac{q}{EJ}\cdot\frac{\ell^4}{24} + \frac{5}{8}\frac{q\ell}{EJ}\cdot\frac{\ell^3}{6} - \frac{1}{8}\frac{q\ell^2}{EJ}\cdot\frac{\ell^2}{2} = 0$

$\frac{-q\ell^4}{EJ\cdot 24} + \frac{5\,q\ell^4}{48EJ} - \frac{q\ell^4}{16EJ} = 0$ verificata (b.c.)

$\varphi_1(0)=0 \quad M_2(\ell)=0 \qquad -\frac{q}{EJ}\cdot\frac{\ell^2}{2} + \frac{5}{8}\frac{q\ell}{EJ}\cdot\ell - \frac{1}{8}\frac{q\ell^2}{EJ} = 0$

verificata

SPOSTAMENTO MASSIMO PER $s=\ell/2$

$v_1(\ell/2) = -\frac{q}{EJ}\cdot\frac{(\ell/2)^4}{24} + \frac{7}{16}\cdot\frac{q\ell}{EJ}\cdot\frac{(\ell/2)^3}{6} - \frac{1}{16}\frac{q\ell^2}{EJ}\cdot\frac{(\ell/2)^2}{2} = \frac{-1}{2}\cdot\frac{q\ell^4}{384EJ}$

verso le basso

$v_2(\ell/2) = -\frac{q}{EJ}\cdot\frac{(\ell/2)^4}{24} + \frac{5}{8}\cdot\frac{q\ell}{EJ}\cdot\frac{(\ell/2)^3}{6} - \frac{1}{8}\frac{q\ell^2}{EJ}\cdot\frac{(\ell/2)^2}{2} = -\frac{1}{3}\cdot\frac{q\ell^4}{64EJ}$

verso le basso

STUDIO SOLLECITAZIONI

$M_1(s) \longrightarrow M_1(0) = C_2 \longrightarrow -\frac{3}{48}\frac{q\ell^2}{EJ} = -\frac{1}{16}\frac{q\ell^2}{EJ}$

$M_1(s) \longrightarrow M_1(\ell) = -\frac{q}{EJ}\cdot\frac{\ell^2}{2} + \frac{7}{16}\cdot\frac{q\ell}{EJ}\cdot\ell - \frac{1}{16}\frac{q\ell^2}{EJ} = -\frac{1}{8}\frac{q\ell^2}{EJ}$

$T_1 = -M'\cdot EJ = qs - \frac{7}{16}q\ell \Rightarrow$

$T_1(0) = -\frac{7}{16}q\ell$

$T_1(\ell) = \frac{9}{16}q\ell$

$V_1(0)=0 \longrightarrow \boxed{C_4=0}$

$\varphi_1(0)=0 \longrightarrow \boxed{C_3=0}$

$V_1(\ell)=0 \longrightarrow -\frac{q}{EJ}\cdot\frac{\ell^2}{12}+C_1\frac{\ell}{3}+C_2=0$

TROVO C_2:

$\boxed{C_2=\frac{q\ell^2}{12EJ}-\frac{C_1\ell}{3}}$ LO SOSTITUISCO IN $M_1(\ell)=M_2(0)$

$M_1(\ell)=M_2(0)=-\frac{q}{EJ}\cdot\frac{\ell^2}{2}+C_1\ell+\frac{q\ell^2}{12EJ}-\frac{C_1\ell}{3}=-\frac{5}{12}\frac{q\ell^2}{EJ}+\frac{2}{3}C_1\ell$

DEDUCO CHE $\boxed{M_1(\ell)=M_2(0)\neq 0}$

SOSTITUISCO C_2 IN $\varphi_1(\ell)=\varphi_2(0)$

$\varphi_1(\ell)=\varphi_2(0)=-\frac{q\ell^3}{6EJ}+\frac{C_1\ell^2}{2}+\frac{q\ell^3}{12EJ}-\frac{C_1\ell^2}{3}=-\frac{q\ell^3}{12EJ}+\frac{C_1\ell^2}{6}$

DEDUCO CHE $\boxed{\varphi_1(\ell)=\varphi_2(0)\neq 0}$

TROVO C_6:

$M_2(\ell)=-\frac{q}{EJ}\cdot\frac{\ell^2}{2}+C_5\ell+C_6=0$

$\boxed{C_6=\frac{q\ell^2}{2EJ}-C_5\ell}$ LO SOSTITUISCO IN $V_2(\ell)$

$V_2(\ell)=0 \quad -\frac{q}{EJ}\cdot\frac{\ell^4}{24}+C_5\frac{\ell^3}{6}+\frac{\ell^2}{2}\left(\frac{q\ell^2}{2EJ}-C_5\ell\right)+C_7\cdot\ell=0$

$\frac{5}{24}\frac{q\ell^3}{EJ}-\frac{C_5}{3}\ell^2+C_7=0$

DA QUI TROVO C_7:

$\boxed{C_7=\frac{C_5}{3}\ell^2-\frac{5}{24}\frac{q\ell^3}{EJ}}$

~~PONGO $C_7=0$~~ e SOSTITUISCO IN $V_2(\ell)$

F.2

$V_2(\ell)=0 \quad \frac{5}{24}\frac{q\ell^3}{EJ}=\frac{C_5}{3}\ell^2 \quad \boxed{C_5=\frac{5}{8}\frac{q\ell}{EJ}}$

SOSTITUISCO C_5 IN C_6

$\frac{q\ell^2}{2}-C_5\ell=C_6 \longrightarrow \frac{q\ell^2}{2}-\frac{5}{8}\frac{q\ell^2}{EJ}=\boxed{-\frac{1}{8}\frac{q\ell^2}{EJ}=C_6}$

RICAPITOLANDO

$C_5=\frac{5}{8}\frac{q\ell}{EJ} \qquad C_6=-\frac{1}{8}\frac{q\ell^2}{EJ} \qquad C_7=0 \qquad C_8=0$

$M_2(0)=C_6=-\frac{1}{8}\frac{q\ell^2}{EJ}$ IO SO CHE $M_2(0)=M_1(\ell)\neq 0$

$M_2(0)=M_1(\ell)=C_6$

$M_1(\ell)=\frac{5}{12}\frac{q\ell^2}{EJ}+\frac{2}{3}C_1\ell=-\frac{1}{8}\frac{q\ell^2}{EJ} \qquad \boxed{C_1=\frac{7}{16}\frac{q\ell}{EJ}}$

TROVO C_2:

$C_2=\frac{q\ell^2}{12EJ}-\frac{7}{16}\cdot\frac{q\ell}{EJ}\cdot\frac{\ell}{3}=\frac{-3}{48}\cdot\frac{q\ell^2}{EJ}$ $\boxed{C_2=-\frac{1}{16}\frac{q\ell^2}{EJ}}$

RICAPITOLANDO

$C_1=\frac{7}{16}\frac{q\ell}{EJ} \qquad C_2=-\frac{1}{16}\frac{q\ell^2}{EJ} \qquad C_3=0 \qquad C_4=0$

F.3

$M_2(s) = \rightarrow M_2(0) = C_6 = -\frac{1}{8} p\ell^2$

$\rightarrow M_2(\ell) = -\frac{q}{EJ} \cdot \frac{\ell^2}{2} + \frac{5}{8} \frac{p\ell}{EJ} \cdot \ell - \frac{1}{8} \frac{p\ell^2}{EJ} = 0$

$T_2 = -M' \cdot EJ = q \cdot s - C_5 = q \cdot s - \frac{5}{8} p\ell \rightarrow T_2(0) = -\frac{5}{8} p\ell$

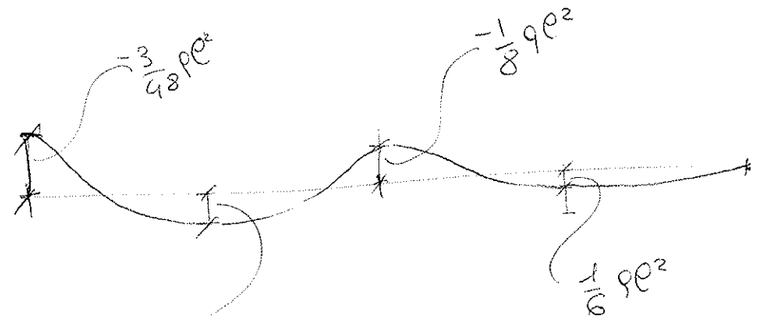
$\rightarrow T_2(\ell) = \frac{3}{8} p\ell$

DISEGNO : GRAFICI

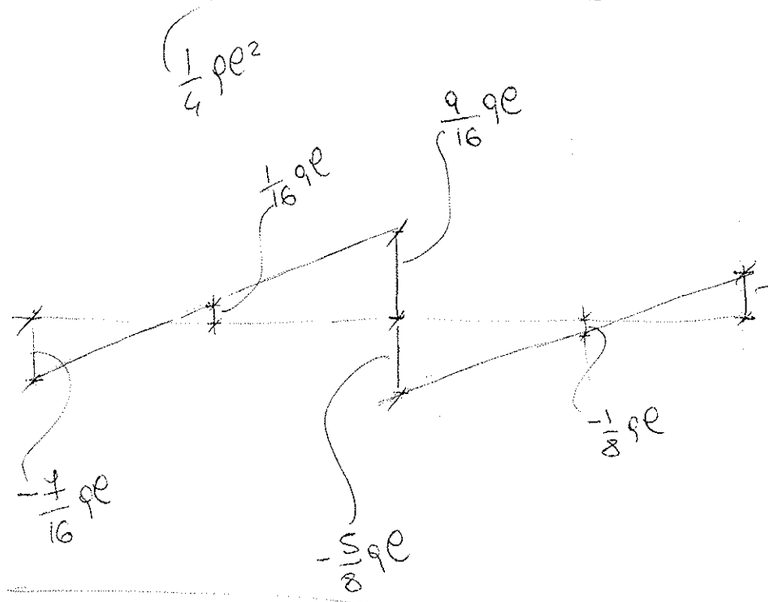
STUDIO A METÀ TRATTO

$M_1(\ell/2) = \frac{1}{4} q\ell^2 / EJ$

$M_2(\ell/2) = \frac{1}{6} p\ell^2 / EJ$



MOMENTO



TAGLIO

$T_1(\ell/2) = \frac{1}{16} q\ell$

$T_2(\ell/2) = -\frac{1}{8} p\ell$

REAZIONI VINCOLARI